

Elastic ρ' and ϕ Meson Photo- and Electroproduction with Non-Resonant Background

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Abstract

The backgrounds due to the direct diffractive dissociation of the photon into the $\pi^+\pi^-$ as well as K^+K^- pairs to the "elastic" diffractive ρ' and ϕ mesons production in electron-proton collisions are calculated. It is shown that the role of background and background-resonant interference contributions is very important in experimental distribution of $M_{\pi^+\pi^-}$ in the ρ' mass region. The amplitude for the background process is proportional to the π -meson - proton or K -meson - proton cross sections. Therefore, describing the HERA data, one can estimate $\sigma(\pi p)$ and $\sigma(Kp)$ at energies higher than the region of direct measurements.

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1 Introduction

It was noted many years ago that the form of the ρ -meson peak is distorted by the interference between resonant and non-resonant $\pi^+\pi^-$ production. For the case of "elastic" ρ^0 photoproduction the effect was studied by P.Söding in [1] and S.Drell [2]. At high energies the main (and the only) source of background is the Drell-Hiida-Deck process [3] (see fig. 1). The incoming photon fluctuates into the pion pair and then πp -elastic scattering takes place. Thus the amplitude for the background may be written in terms of the pion-proton cross section. It was demonstrated [4, 5] that the interference with some non-resonant background is indeed needed to describe the distribution over the mass - M of $\pi^+\pi^-$ pair. M.Arneodo proposed [6] that this effect can be used to estimate the value of $\sigma_{\pi p}$ from HERA data at high energies, in the range which is not otherwise acceptable. Our calculations [7] show that really the data on M distributions in the case of $\pi^+\pi^-$ pair production can be described satisfactory with the realistic value of $\sigma_{\pi p}^{tot} \sim 30$ mb at energy $s_{\pi p} \sim (2 \div 3)10^3$ GeV².

To prove that this method of extraction of total πp cross section is really working, it should give the same values of $\sigma_{\pi p}^{tot}$ for all resonances which decay into $\pi^+\pi^-$ pair. So in the present paper we will consider the mass distribution of produced $\pi^+\pi^-$ in the region of ρ' meson. The situation with $\pi^+\pi^-$ pair production in the region of ρ' mass is not simple because there are evidence [8] that here two different resonances exist, $\rho(1450)$ and $\rho(1700)$ and the "tail" from $\rho(770)$ production is not negligibly small. However, if the amplitude of $\pi\pi$ scattering is an analytical function with good behaviour at infinity in $M_{\pi\pi}$ complex plane, the $M_{\pi\pi}$ distribution should be determined totally by the singularities of elastic $\pi\pi$ amplitude, i.e. by contributions of all resonances and the cut corresponding to the 2π non-resonant production.

In Sect. 2 the formulae for the 2π background which are valid for the DIS as well as for the photoproduction region are presented. The expression differs slightly from the Söding's one as we take into account the meson form factor and the fact that one meson propagator is off-mass shell. We consider also the absorption correction coming from the diagram where both mesons (π^+ and π^- , or K^+ and K^-) directly interact with the target proton. The role of the interference in ρ' photo- and electroproduction is discussed in Sect. 3. We will also consider in Sect. 4 the case of ϕ photo- and electroproduction which allows one, in principle, to extract the value of σ_{Kp}^{tot} at high energies.

2 Production amplitudes

The cross section of vector meson photo- and electroproduction may be written as:

$$\frac{d\sigma^D}{dM^2 dt} = \int d\Omega |A_r. + A_{n.r.}|^2, \quad (1)$$

where $A_r.$ and $A_{n.r.}$ are the resonant and non-resonant parts of the production amplitude, $D = L, T$ for longitudinal and transverse photons, $t = -q_t^2$ is the momentum transferred to the proton and $d\Omega = d\phi d\cos(\theta)$, where ϕ and θ are the azimuthal and polar angles between the π^+ and the proton direction in the 2π rest frame.

2.1 Amplitude for resonant production

We will use the simple phenomenological parametrization of the production amplitude because our main aim is the discussion of the interference between resonant and non-resonant contributions. So the amplitude for resonant process $\gamma p \rightarrow V p$:

$$A_V = \sqrt{\sigma_V} e^{-b_V q_t^2/2} \frac{\sqrt{M_0 \Gamma}}{M^2 - M_0^2 + i M_0 \Gamma} \frac{Y^D(\theta, \phi)}{\sqrt{\pi}}, \quad (2)$$

where M_0 and $\Gamma = \Gamma_0$ are the mass and the width of vector meson; b_V is the t -slope of the "elastic" V production cross section $\sigma_V \equiv d\sigma(\gamma p \rightarrow V p)/dt$ (at $t = 0$) and the functions $Y^D(\theta, \phi)$, $D = T, L$ describe the angular distribution of the pions produced through the vector meson decay:

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta, \quad (3)$$

$$Y_1^{\pm 1} = \sqrt{\frac{3}{8\pi}} \sin\theta \cdot e^{\pm i\phi}. \quad (4)$$

Note that for transverse photons with polarization vector \vec{e} one has to replace the last factor $e^{\pm i\phi}$ in eq. (4) by the scalar product $(\vec{e} \cdot \vec{n})$, where \vec{n} is the unit vector in the pion transverse momentum direction.

2.2 Amplitude for non-resonant production

The amplitude for the non-resonant process $\gamma p \rightarrow \pi^+ \pi^- p$ is:

$$A_{n.r.} = \sigma_{\pi p} F_\pi(Q^2) e^{bt/2} \frac{\sqrt{\alpha}}{\sqrt{16\pi^3}} B^D \sqrt{z(1-z) \left| \frac{dz}{dM^2} \right| \left(\frac{M^2}{4} - m_\pi^2 \right) |\cos\theta|}, \quad (5)$$

where b is the t -slope of the elastic πp cross section, $F_\pi(Q^2)$ is the pion electromagnetic form factor ($Q^2 = |q_\gamma^2| > 0$ is the virtuality of the incoming photon), $\alpha = 1/137$ is the electromagnetic coupling constant and z – the photon momentum fraction carried by the π^- -meson; $\sigma_{\pi p}$ is the total pion-proton cross section.

The factor B^D is equal to

$$B^D = \frac{(e_\mu^D \cdot k_{\mu-}) f(k_-'^2)}{z(1-z)Q^2 + m_\pi^2 + k_{t-}^2} - \frac{(e_\mu^D \cdot k_{\mu+}) f(k_+'^2)}{z(1-z)Q^2 + m_\pi^2 + k_{t+}^2} \quad (6)$$

For longitudinal photons the products $(e_\mu^L \cdot k_{\mu\pm})$ are: $(e_\mu^L \cdot k_{\mu-}) = z\sqrt{Q^2}$ and $(e_\mu^L \cdot k_{\mu+}) = (1-z)\sqrt{Q^2}$, while for the transverse photons we may put (after averaging) $e_\mu^T \cdot e_\nu^T = \frac{1}{2}\delta_{\mu\nu}^T$.

Expressions (5) and (6) are the result of straightforward calculation of the Feynman diagram fig. 1. The first term in (6) comes from the graph fig. 1 (in which the Pomeron couples to the π^+) and the second one reflects the contribution originated by the $\pi^- p$ interaction. The negative sign of π^- electric charge leads to the minus sign of the second term. We omit here the phases of the amplitudes. In fact, the common phase is inessential for the cross section, and we assume that the relative phase between $A_r.$ and $A_{n.r.}$ is small (equal to zero) as in both cases the phase is generated by the same 'Pomeron' ¹ exchange.

The form factor $f(k'^2)$ is written to account for the virtuality ($k'^2 \neq m_\pi^2$) of the t -channel (vertical solid line in fig. 1) pion. As in fig. 1 we do not deal with pure elastic pion-proton scattering, the amplitude may be slightly suppressed by the fact that the incoming pion is off-mass shell. To estimate this suppression we include the form factor (chosen in the pole form)

$$f(k'^2) = 1/(1 + k'^2/m^2). \quad (7)$$

¹Better to say – 'vacuum singularity'.

The same pole form was used for $F_\pi(Q^2) = 1/(1 + Q^2/m_\rho^2)$. In the last case the parameter m_ρ is the mass of the ρ -meson, but the value of m' (in $f(k'^2)$) is expected to be larger. In the case of $\pi^+\pi^-$ production it should be of the order of mass of the next resonance from Regge π -meson trajectory; i.e. it should be the mass of $\pi(1300)$ or $b_1(1235)$. Thus we put $m'^2 = 1.5 \text{ GeV}^2$.

Finally we have to define k'_\pm and $k_{t\pm}$.

$$\vec{k}_{t-} = -\vec{K}_t + z\vec{q}_t \quad \vec{k}_{t+} = \vec{K}_t + (1-z)\vec{q}_t \quad (8)$$

and

$$k'^2_- = \frac{z(1-z)Q^2 + m_\pi^2 + k_{t-}^2}{z}, \quad k'^2_+ = \frac{z(1-z)Q^2 + m_\pi^2 + k_{t+}^2}{1-z}. \quad (9)$$

In these notations

$$M^2 = \frac{K_t^2 + m_\pi^2}{z(1-z)}, \quad dM^2/dz = (2z-1)\frac{K_t^2 + m_\pi^2}{z^2(1-z)^2}$$

and $z = \frac{1}{2} \pm \sqrt{1/4 - (K_t^2 + m_\pi^2)/M^2}$ with the pion transverse (with respect to the proton direction) momentum \vec{K}_t (in the 2π rest frame) given by expression $K_t^2 = (M^2/4 - m_\pi^2)\sin^2\theta$. Note that the positive values of $\cos\theta$ correspond to $z \geq 1/2$ while the negative ones $\cos\theta < 0$ correspond to $z \leq 1/2$.

2.3 Absorptive correction

To account for the screening correction we have to consider the diagram fig. 2, where both pions interact directly with the target. Note that all the rescatterings of one pion (say π^+ in fig. 1) are already included into the $\pi\pi$ elastic amplitude. The result may be written in form of eq. (5) with the new factor \tilde{B}^D instead of the old one $B^D = B^D(\vec{K}_t, \vec{q})$:

$$\tilde{B}^D = B^D(\vec{K}_t, \vec{q}) - \int C \frac{\sigma_{\pi p} e^{-bl_t^2}}{16\pi^2} B^D(\vec{K}_t - z\vec{l}_t, \vec{q}) d^2l_t \quad (10)$$

where the second term is the absorptive correction (fig. 2) and l_μ is the momentum transferred along the 'Pomeron' loop. The factor $C > 1$ reflects the contribution of the enhancement graphs with the diffractive excitation of the target proton in intermediate state. In accordance with the HERA data [9], where the cross section of "inelastic" (i.e. with the proton diffracted) ρ photoproduction was estimated as $\sigma^{inel} \simeq 0.5\sigma^{el}$ we choose $C = 1.5 \pm 0.2$.

3 Elastic ρ' photo- and electroproduction cross section

The situation with resonance ρ' is not clear. The enhancement in the four pion invariant mass distribution at ~ 1.6 GeV which was considered as $\rho'(1600)$ particle now can be considered [8] as two interfering resonances, $\rho(1450)$ and $\rho(1700)$ (r_1 and r_2 in the formulae below) with masses 1.465 GeV and 1.70 GeV and widths 0.31 GeV and 0.235 GeV, respectively. However the presented formulae are valid for one-resonant production. The relative phase of the amplitudes of these resonances production is practically unknown. For these reason we will present the calculated results for two cases, one resonance with $M = 1.57$ GeV and $\Gamma = 180$ MeV [12], using eq. (1), and two-resonances with the masses and widths presented above and with zero relative phase. In the last case we will use expression

$$\frac{d\sigma^D}{dM^2 dt} = \int d\Omega | - A_{r_1} - A_{r_2} + A_{n.r.} |^2 \quad (11)$$

instead of eq. (1) and in agreement with paper [10] we use the negative sign of the resonant contribution that can be connected with the negative signs of $r_1, r_2 \rightarrow \pi\pi$ decay amplitude. The main aim of our calculations is to demonstrate the role of background contribution to the resulting $M_{2\pi}$ distribution.

The results at $Q^2 = 0$ (photoproduction) for the cases of one-resonance and two-resonances production are shown in figs. 3a and 3b. Both slope parameters, b_ρ in eq.(2) and b in eq.(5) were assumed to be equal to 10 GeV⁻². The dashed curves correspond to the resonant production contributions. They are normalized to the experimental ρ' to ρ ratio in the two-pion final state (i.e. the product of cross section ratio and $\rho' \rightarrow \pi^+\pi^-$ branching) that is equal to 0.0134 ± 0.0023 at $Q^2 = 0$ [10] and we assume that in the case of two-resonances production their production cross sections are equal. The values of two-pion branching ratios for these states were assumed to be 0.07 and 0.25, respectively [8].

The contributions of non-resonant production (dotted curves) and its interference with resonant one (dash-dotted curves) depend on the form factor $f(k'^2)$, the screening corrections and the value of $\sigma_{\pi p}$. Here the form factor was used in the form eq.(7) with $m'^2 = 1.5$ GeV², screening correction in

the form eq.(10) and the value of $\sigma_{\pi p} = 31$ mb were used ². The sum of all three contributions are presented by solid curves. One can see that the contributions of non-resonant and interference production dominate for the mass region of $\rho(1450)$ and the experimental enhancement in the two-pion mass distribution here is connected mainly with background and interference contributions as well as with possible "tails" from $\rho(770)$ and $\rho(1700)$ production. This enhancement should be sensitive to the value of $\sigma_{\pi p}$.

To demonstrate the role of the relative sign of resonant and non-resonant (background) production we present in figs. 3c and 3d the calculations with the positive sign of resonant contribution in eq. (11). Now we present in fig. 3d the sum of two resonances and their interference by dashed curve, the sum of all resonant-background contributions by dash-dotted curve and background contribution by dotted curve. One can see that the shape of the summary curves (solid ones) are changed drastically in comparison with the cases of figs. 3a and 3b, especially in the case of two-resonant production.

We have considered also the process of $\pi^+\pi^-$ production in the ρ' mass region with accounting the "tail" of $\rho(770)$ contribution, i.e. using the expression

$$\frac{d\sigma^D}{dM^2 dt} = \int d\Omega |A_\rho - A_{r1} - A_{r2} + A_{n.r.}|^2 . \quad (12)$$

It is interesting that the contributions of $\rho(770)$ Breit-Wigner peak and its interference with background practically cancel at the used parameters (on the level of $\sim 90\%$) the background contribution shown in figs. 3. It means that $\pi^+\pi^-$ production in the mass region, say 1.2-1.9 GeV is determined practically only by one or two ρ' resonances ³.

At very large Q^2 the background amplitude eq.(5) becomes negligible as, even without the additional form factor (i.e. at $f(k'^2) \equiv 1$), the non-resonance cross section falls down as $1/Q^8$ ⁴.

The results of our calculations of $d\sigma/dM$ for $Q^2 = 10$ GeV² using eq. (1) for one ρ' resonance and eq. (11) for two resonances with form factor,

²ZEUS analysis [11] of $\pi^+\pi^-$ production in the ρ mass region of elastic photoproduction.

³As the accuracy of this cancellation depends crucially on the values of used parameters, one has to check the "background" contribution experimentally, say in the region of $M \sim 1$ GeV.

⁴In the amplitude $A_{n.r.}$ one factor $1/Q^2$ comes from the electromagnetic form factor $F_\pi(Q^2)$ and another one – from the pion propagator (term - $z(1-z)Q^2$ in the denominator of B^D (see eq.(6)).

eq.(7), $\sigma_{\pi p} = 31$ mb and with screening correction are presented in figs. 4a and 4b, respectively. They are normalized to the experimental [12] ρ' to ρ ratios ($0.36 \pm 0.07 \pm 0.11$ for $\pi^+\pi^-\pi^+\pi^-$ channel) which were assumed to be equal to 0.8 for one $\rho(1570)$ resonance and to 0.4 at for both states, $\rho(1450)$ and $\rho(1700)$. The pure resonant contributions (the sum of two resonances and their interference in the case of fig. 4b) are shown by dashed curves (we assume that the longitudinal contribution to the resonant production is equal to the transverse one) and one can see that in both cases resonant production dominate.

The contribution of $\rho(770)$ here is small. For $M_{\pi\pi} \sim 1.4$ GeV the $\rho(770)$ "tail" is 5-6 times smaller than the non-resonant $\pi^+\pi^-$ background.

4 Elastic ϕ photo- and electroproduction cross section

In the case of ϕ -meson photo- and electroproduction the corrections to the pure resonant production should be small because the resonant peak is very narrow. However their contribution should be investigated.

We will consider the channel $\phi \rightarrow K^+K^-$ with branching ratio $49.1 \pm 0.6\%$. The absolute normalization of the ϕ photoproduction cross section can be obtained by integration of K^+K^- mass distribution near the ϕ peak that should corresponds to the value $\sigma_{\gamma p \rightarrow \phi p} = 0.96 \pm 0.19_{-18}^{+21} \mu\text{b}$ [13]. The value of b_ϕ parameter equal to 7 GeV^{-2} in eq. (2) for t -slope of the elastic ϕ photoproduction was used in agreement with data of [13]. Here the form factor $f(k'^2)$, was used in the form eq.(7) with $m'^2 = 2 \text{ GeV}^2$, that corresponds to the mass of the next resonance from Regge K -meson trajectory. We take the value $b = 9 \text{ GeV}^{-2}$ for t -slope in elastic Kp cross section and $\sigma_{Kp}^{tot} = 24$ mb for the total cross section of Kp interactions.

The results of calculations for the case of K^+K^- photoproduction are shown in fig. 5a. Of course, resonant contribution dominates in the region of ϕ peak, however some interference contribution can be seen at the right-hand side of the "tail". So in principle it is possible to estimate total Kp cross section, however for this problem it is necessary to have very good identification of secondary kaons ⁵.

⁵Now the main source of experimental background for $\gamma p \rightarrow \phi p$ reaction is $\gamma p \rightarrow \rho p$

The results for K^+K^- electroproduction cross section at $Q^2 = 10 \text{ GeV}^2$ in the mass region of ϕ peak is shown in fig. 5b. Here we assumed that the longitudinal contribution to the resonant production is 1.5 times larger than the transverse one and one can see that the resonance production dominate.

5 Conclusion

We presented simple formulae for the background to elastic ρ' -meson and ϕ photo- and electroproduction which account for the absorptive correction and virtuality of the pion and kaon. The role of resonance – background interference is very significant for the case of $\pi^+\pi^-$ pair photoproduction in the mass region of ρ' -meson. Due to some occasional reasons the "tail" of $\rho(770)$ is practically cancelled by background contribution. At large Q^2 , say at $Q^2 \sim 10 \text{ GeV}^2$, for the case of ρ' production the $\rho(770)$ "tail" is rather small and the main background comes from non-resonant $\pi^+\pi^-$ production. For the case of ϕ production background is small enough.

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process [13].

Figure captions

Fig. 1. Feynman diagram for the two pion photo-(electro)production.

Fig. 2. Diagram for the absorbtive correction due to both pions (kaons) rescattering.

Fig. 3. Resonant (Breit-Wigner, dashed curves), non-resonant background (dotted curves) and their interference (dash-dotted curves) contributions to $\gamma p \rightarrow \pi^+ \pi^- p$ photoproduction process for the cases of one (a,c) and two (b,d) resonances in the ρ' mass region. The cases of figs. a, b and c,d have different signs of the interference contributions, see text. The sums of all contributions are shown by solid curves.

Fig. 4. Resonant (Breit-Wigner, dashed curves), non-resonant background (dotted curves) and their interference (dash-dotted curves) contributions to $\gamma p \rightarrow \pi^+ \pi^- p$ electroproduction process at $Q^2 = 10 \text{ GeV}^2$ for the cases of one (a) and two (b) resonances in the ρ' mass region. The sums of all contributions are shown by solid curves.

Fig. 5. Resonant (Breit-Wigner, dashed curve), non-resonant background (dotted curve) and their interference (dash-dotted curve) contributions to $\gamma p \rightarrow K^+ K^- p$ photoproduction process. The sums of all contributions are shown by solid curve (a). The case of elastic $K^+ K^-$ electroproduction at $Q^2 = 10 \text{ GeV}^2$ (solid curve); the dashed curve presents the resonant contribution (b).

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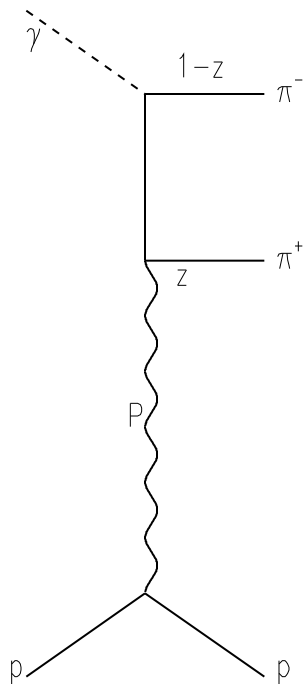


Fig. 1

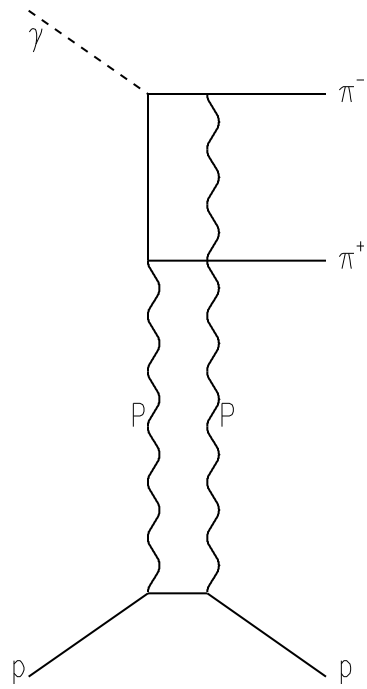


Fig. 2

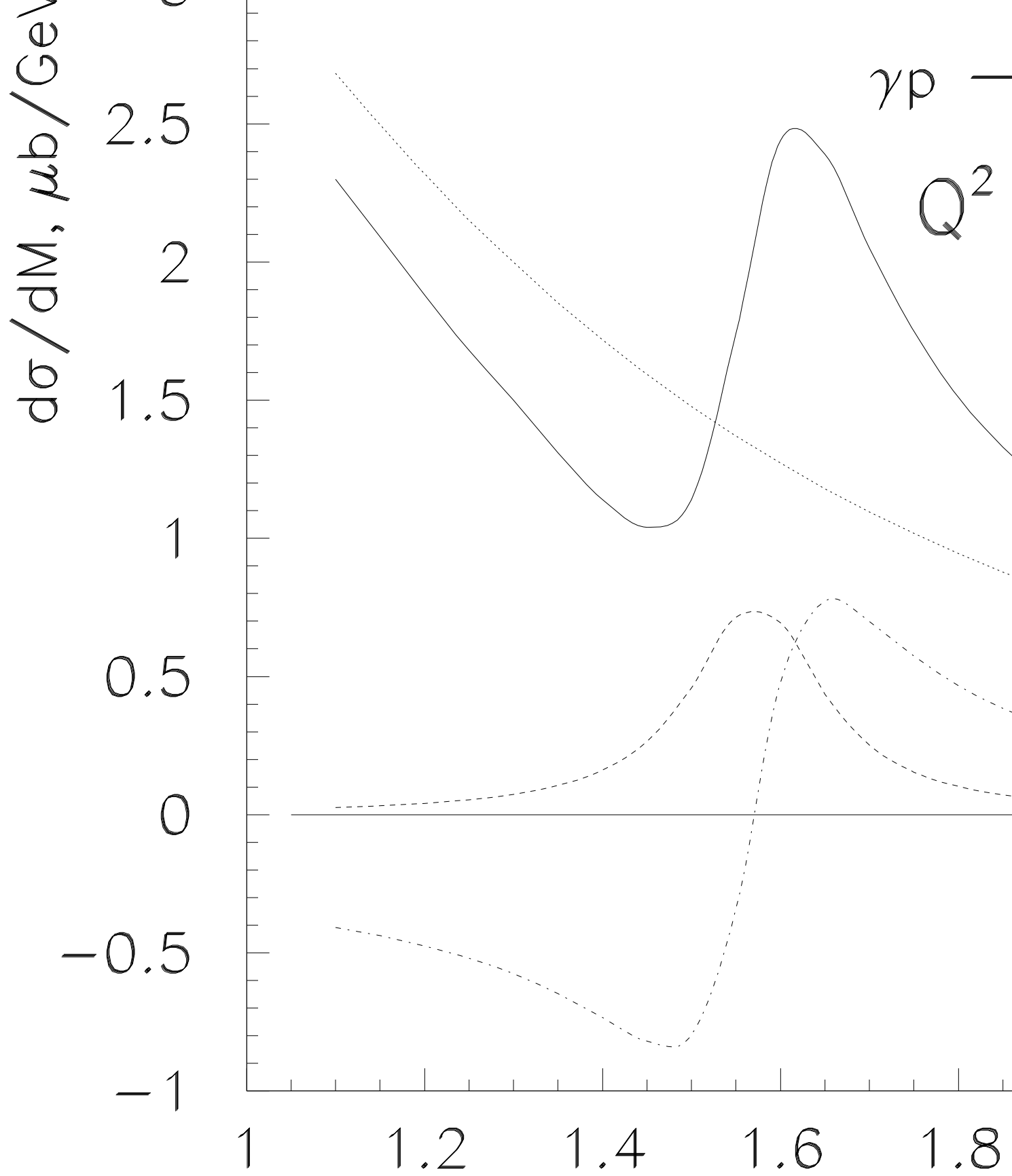


Fig. 3a

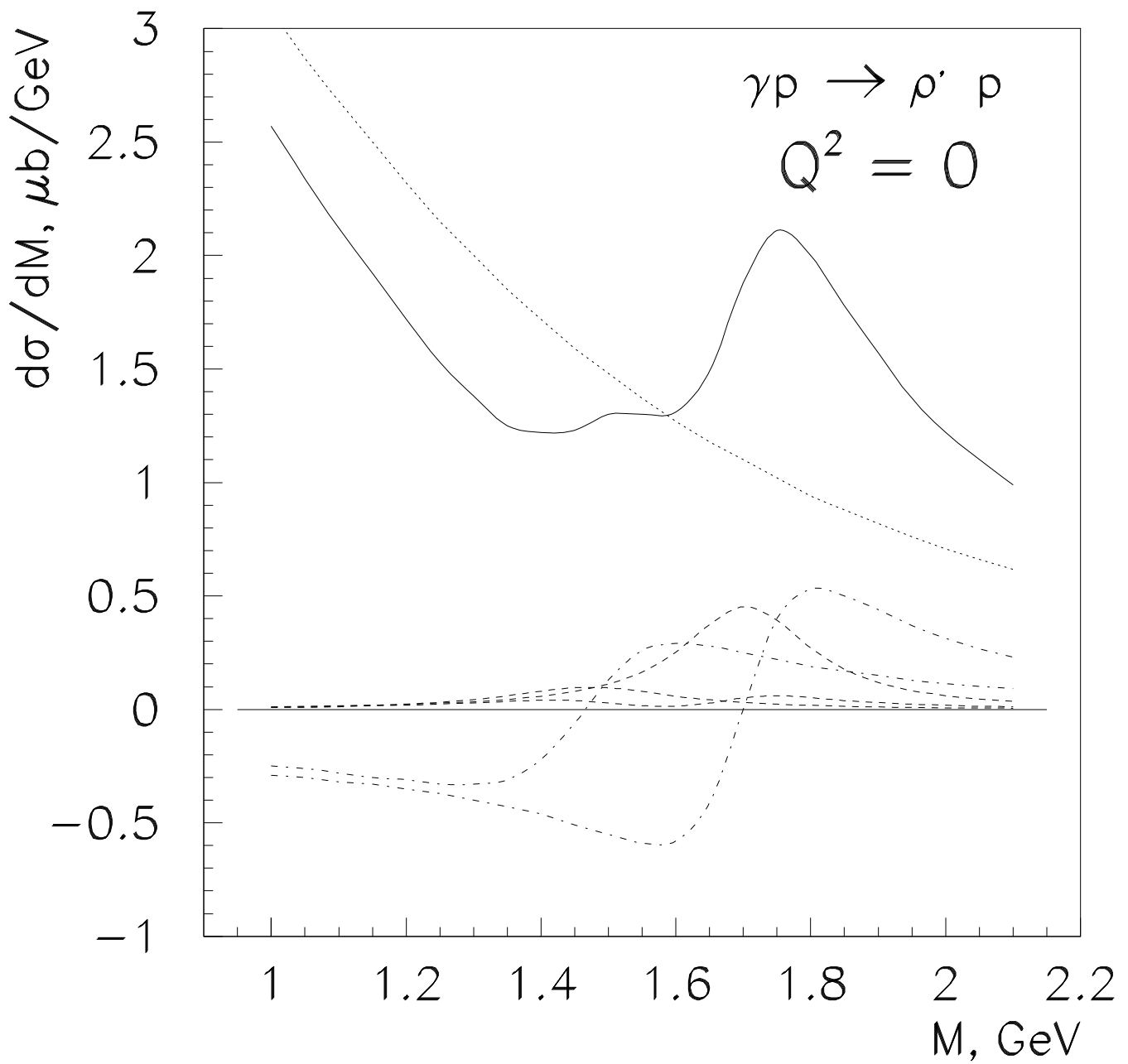


Fig. 3b

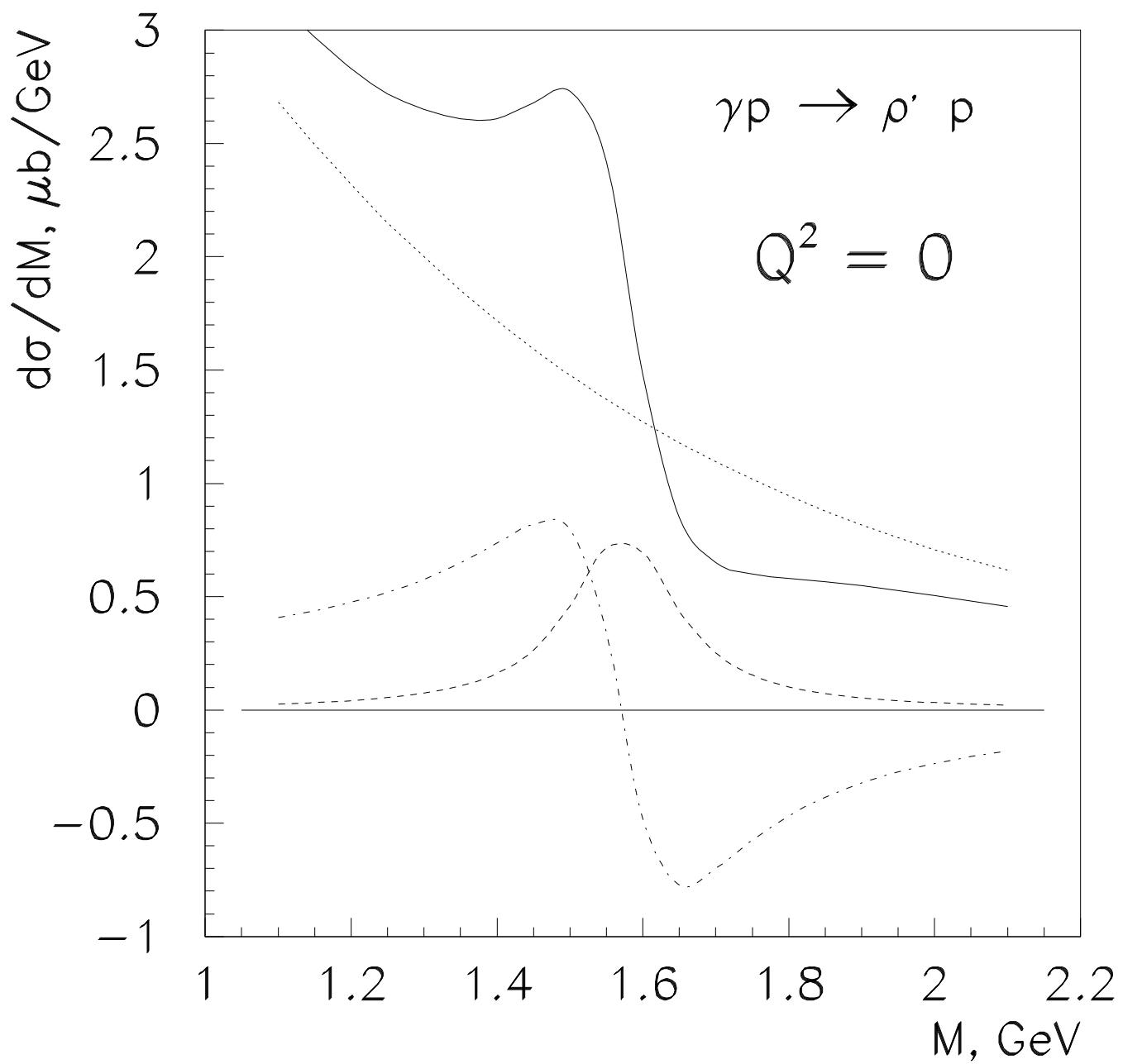


Fig. 3c

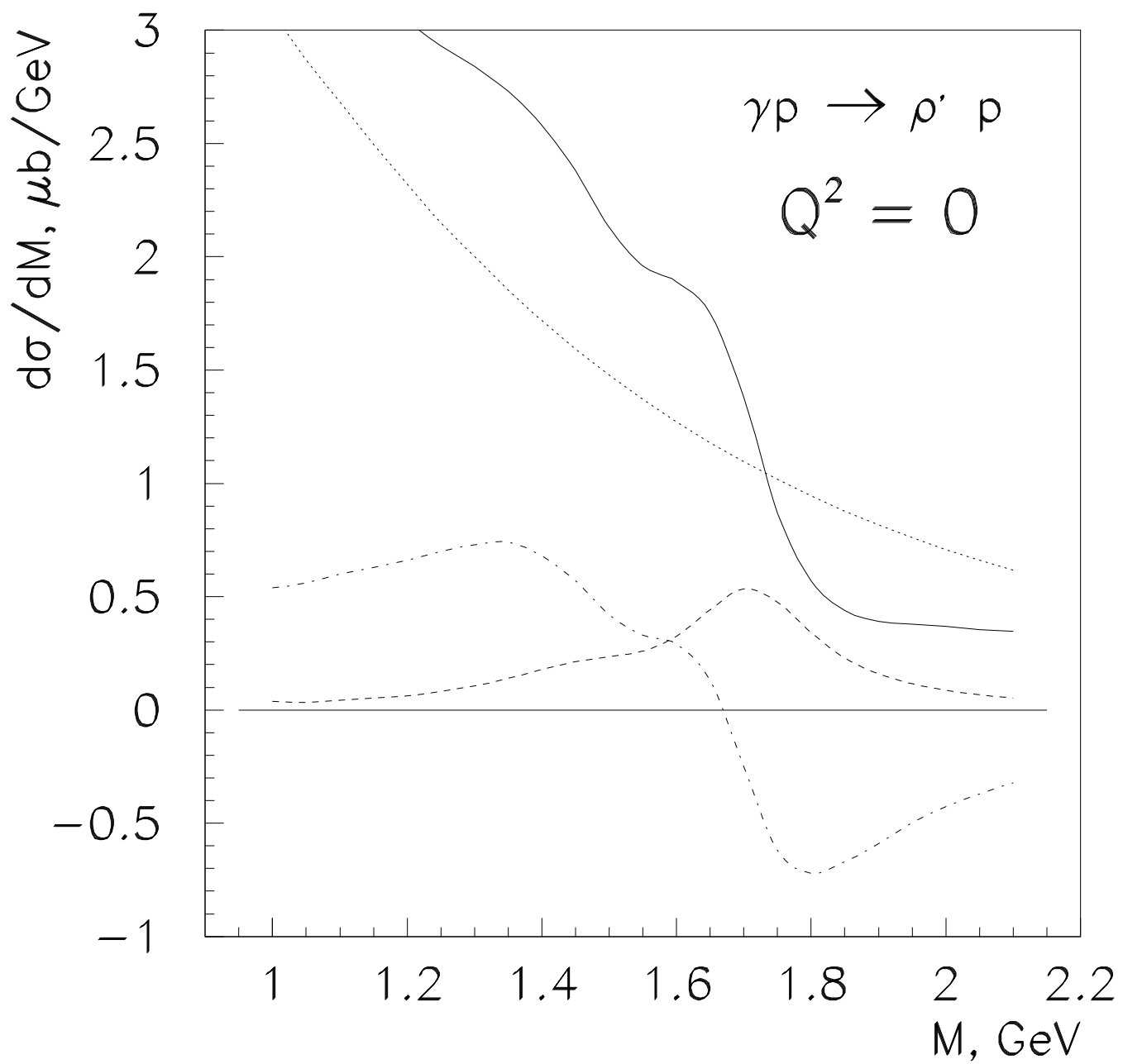


Fig. 3d

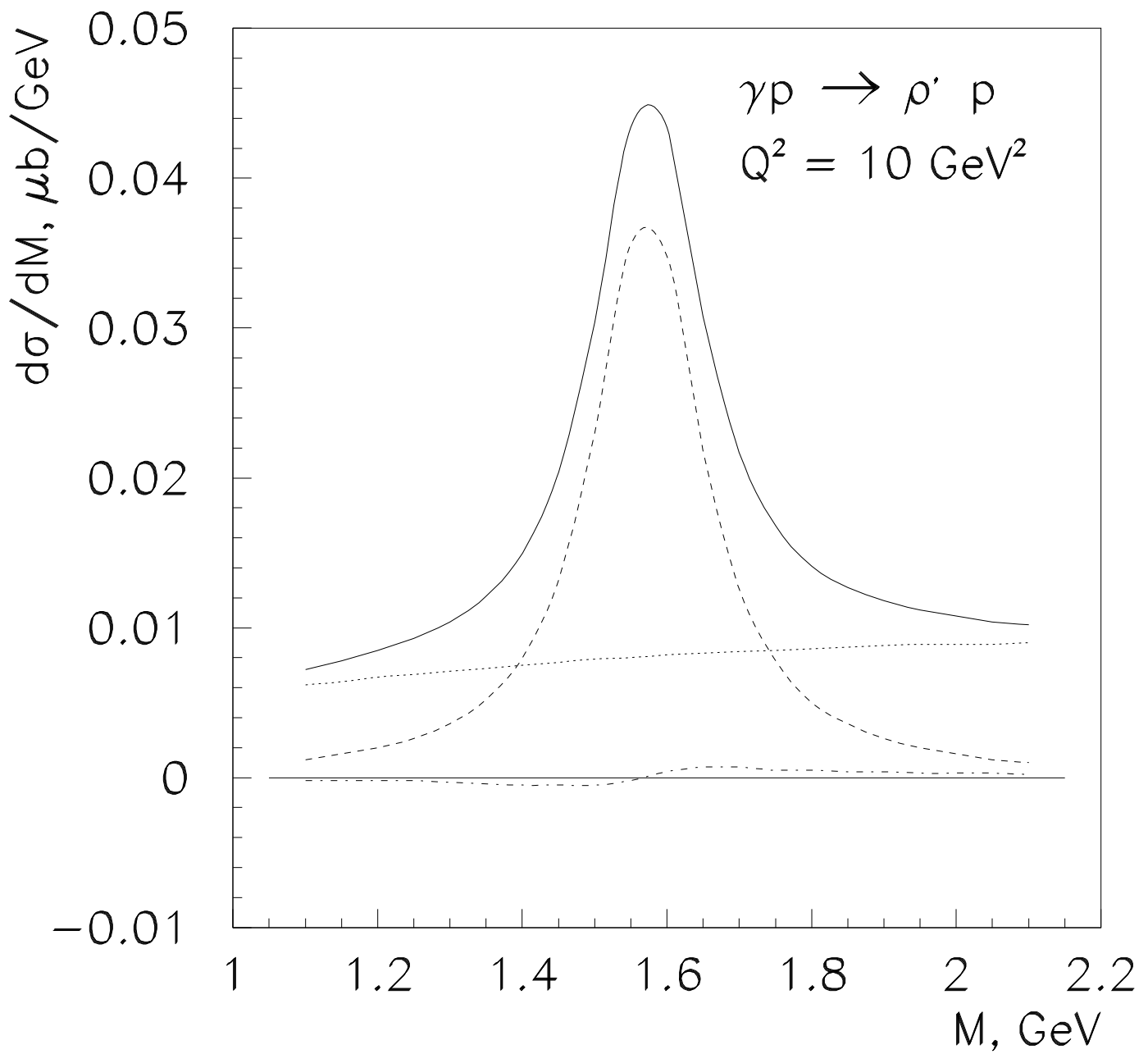


Fig. 4a

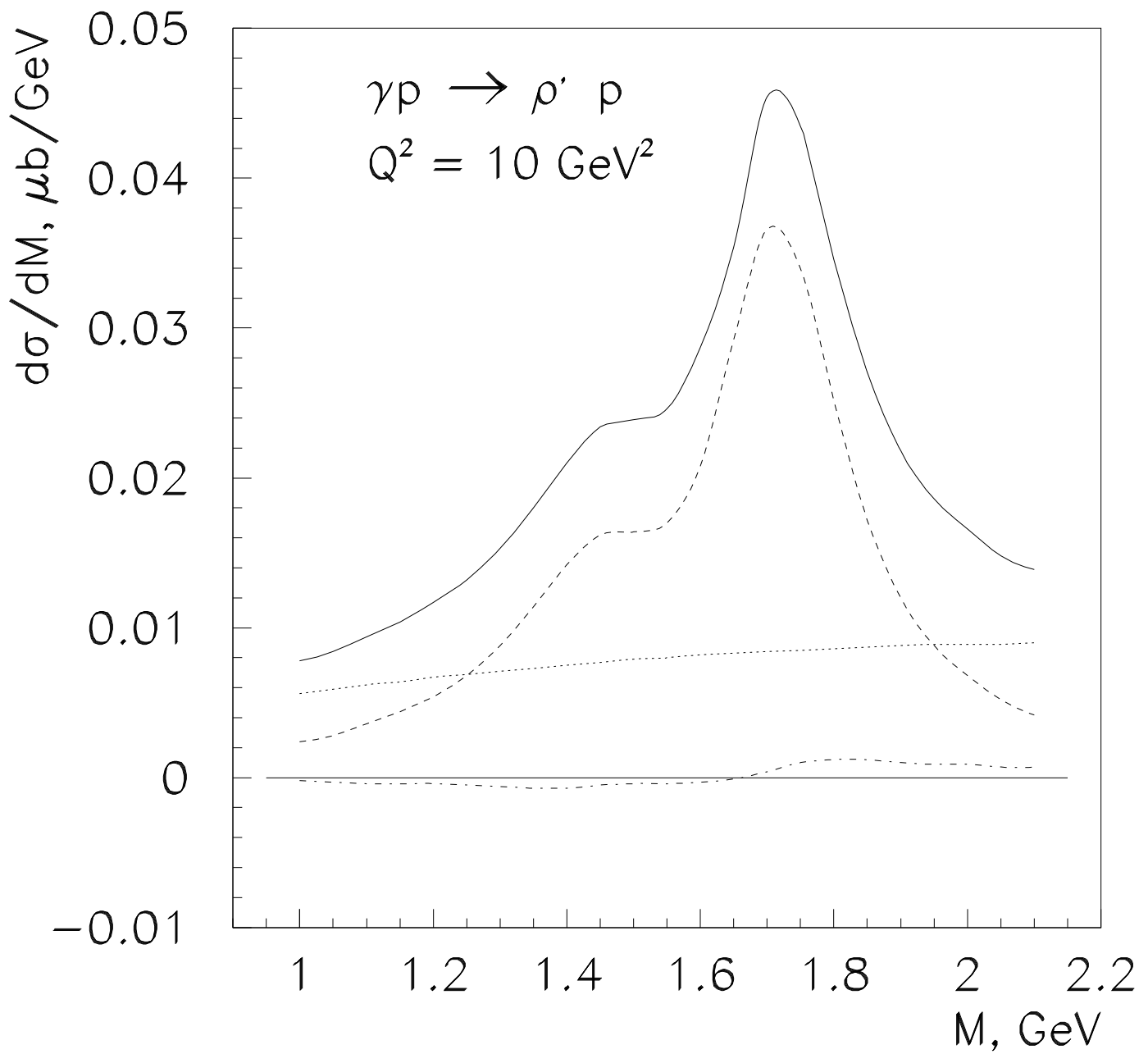


Fig. 4b

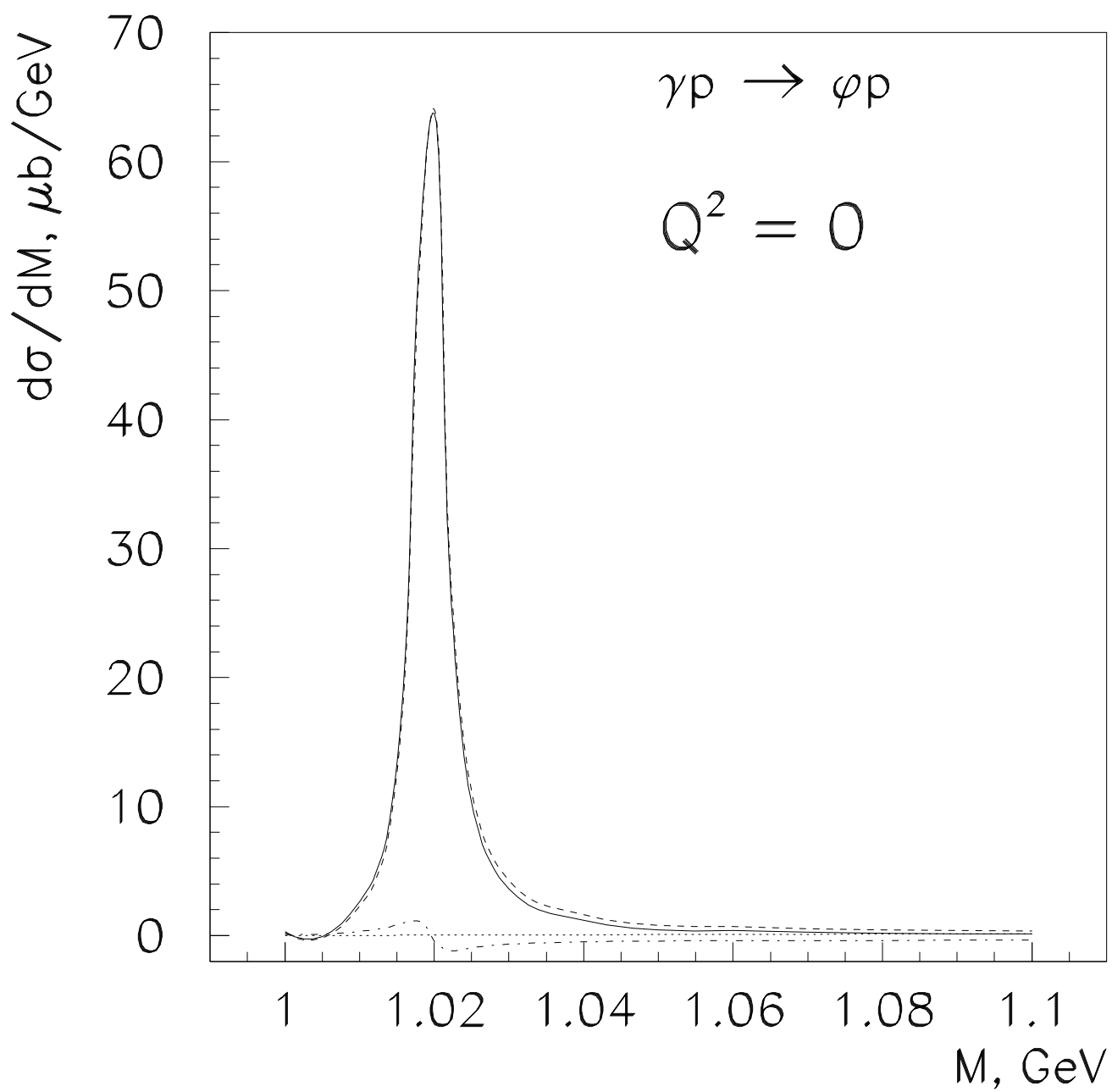


Fig. 5a

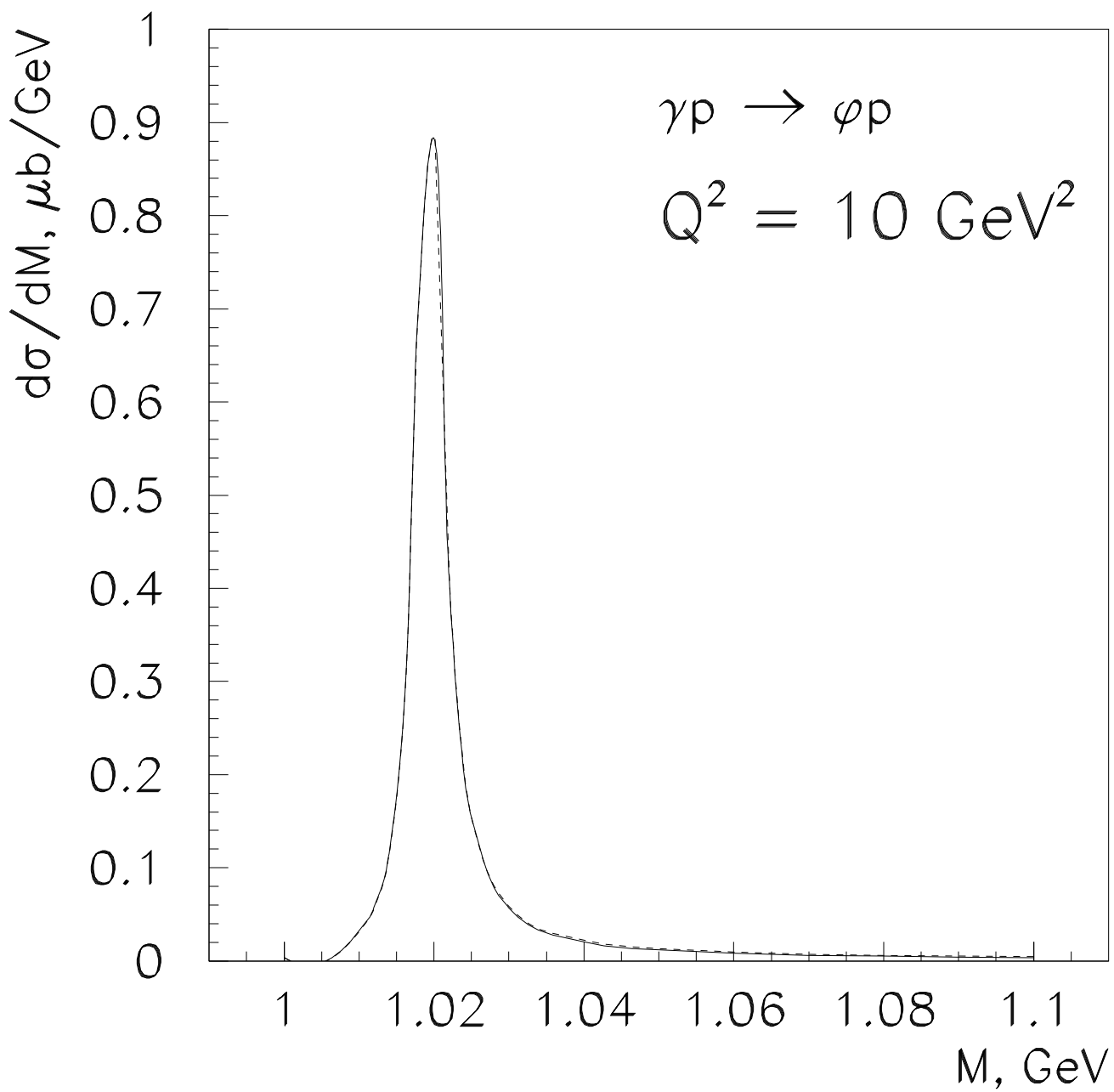


Fig. 5b